



MINISTRY OF EDUCATION, SINGAPORE  
in collaboration with  
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION  
General Certificate of Education Advanced Level  
Higher 2

## MATHEMATICS

9758/02

Paper 2

For examination from 2025

SPECIMEN PAPER

3 hours

Additional Materials: List of Formulae and Results (MF27)

*Suggested Solutions .*

### READ THESE INSTRUCTIONS FIRST

Answer **all** questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.



Singapore Examinations and Assessment Board



Cambridge Assessment  
International Education

## Section A: Pure Mathematics [40 marks]

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1(a)(i)  | <p>The function <math>f</math> is defined as follows.</p> $f: x \mapsto x^2 + 6x - 10, \quad x \in \mathbb{R}$ <p>By using the method of completing the square, find the range of <math>f</math>. [2]</p> <p> <math display="block">x^2 + 6x - 10 = \left[ x^2 + 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 \right] - 10</math> <math display="block">= (x+3)^2 - 19</math> </p> <p>Since the coefficient of <math>x^2</math> is positive, <math>f(x)</math> has a min value<br/> <math>\therefore</math> Range of <math>f = [-19, \infty)</math></p>                                                                                           |
| 1(a)(ii) | <p>Sketch the graph of <math>f</math>. [2]</p> <p> <math>-7.36</math><br/> or <math>-3 - \sqrt{19}</math> </p> <p> <math>-3</math><br/> <math>(-3, -19)</math> </p> <p> <math>-10</math><br/> <math>1.36</math><br/> or <math>-3 + \sqrt{19}</math> </p> <p> <math>y</math><br/> <math>x</math><br/> <math>y = f(x)</math> </p> <p>     1m - correct shape and turning point<br/>     1m - correct x- and y-intercepts.   </p>                                                                                                                                                                                                                                    |
| 1(b)(i)  | <p>The function <math>g</math> is defined as follows.</p> $g: x \mapsto x^2 + 6x - 10, \quad x \leq \alpha$ <p>You are given that the function <math>g^{-1}</math> exists.</p> <p>State the largest possible value of <math>\alpha</math>, explaining why this is the largest value. [2]</p> <p> <math>\alpha = -3</math>. This is because <math>g</math> has a turning point at <math>x = -3</math>. For <math>g^{-1}</math> to exist, <math>g</math> must be one-one. If <math>\alpha &gt; -3</math>, the domain includes points on both sides of the turning point, so <math>g</math> is not one-one. Hence, <math>\alpha = -3</math> is largest value.   </p> |
| 1(b)(ii) | <p>Find an expression for <math>g^{-1}(x)</math>. [2]</p> <p> <math>Let\ y = x^2 + 6x - 10</math><br/> <math>x^2 + 6x - (10+y) = 0</math><br/> <math>x = \frac{-6 \pm \sqrt{6^2 + 4(1)(10+y)}}{2(1)}</math><br/> <math>x = -3 \pm \sqrt{9 + 10 + y}</math> </p> <p> <math>x = -3 \pm \sqrt{y+19}</math><br/> <math>x = -3 - \sqrt{y+19}</math> since <math>x \leq -3</math><br/> <math>\therefore g^{-1}(x) = -3 - \sqrt{x+19}</math> </p>                                                                                                                                                                                                                        |

|                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|--------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>2(a)</b></p> | <p>The first four terms of a sequence of numbers are 4, 2, 2 and 4. The sum of the first <math>n</math> terms of this sequence is <math>S_n</math>.</p> <p>Explain why <math>S_n</math> cannot be a quadratic polynomial in <math>n</math>. [2]</p> <p>Assume <math>S_n</math> is a quadratic polynomial such that <math>S_n = an^2 + bn + c</math><br/> Then <math>S_1 = a + b + c = 4</math> — (1)      <math>S_n = 2n + 2</math> which is not quadratic. Also,<br/> <math>S_2 = 4a + 2b + c = 6</math> — (2)      <math>S_4 = 2(4) + 2 = 10 \neq 12</math>.<br/> <math>S_3 = 9a + 3b + c = 8</math> — (3)<br/> Using the G.C., <math>a = 0, b = 2, c = 2</math> → Hence, <math>S_n</math> cannot be a quadratic polynomial in <math>n</math>.</p> |
| <p><b>2(b)</b></p> | <p>It is given that <math>S_n = an^3 + bn^2 + cn + d</math>.</p> <p>Find the values of <math>a, b, c</math> and <math>d</math>. [4]</p> <p> <math>S_1 = a + b + c + d = 4</math> — (1)<br/> <math>S_2 = 8a + 4b + 2c + d = 6</math> — (2)<br/> <math>S_3 = 27a + 9b + 3c + d = 8</math> — (3)<br/> <math>S_4 = 64a + 16b + 4c + d = 12</math> — (4) </p> <p>Using the G.C.,<br/> <math>a = \frac{1}{3}, b = -2, c = \frac{17}{3}, d = 0</math> //</p> <p>3 marks if all four equations correct.<br/> Deduct 1 mark for every incorrect eq<sup>n</sup>.<br/> 1 mark for <math>a, b, c, d</math> all correct.</p>                                                                                                                                      |

2(c)

Find an expression in terms of  $n$  for the  $n$ th term of the sequence.

[3]

$$S_n = \frac{1}{3}n^3 - 2n^2 + \frac{17}{3}n$$

$n$ th term of sequence,  $T_n$

$$= S_n - S_{n-1}$$

$$= \frac{1}{3}n^3 - 2n^2 + \frac{17}{3}n - \left[ \frac{1}{3}(n-1)^3 - 2(n-1)^2 + \frac{17}{3}(n-1) \right]$$

Applied  
 $S_n - S_{n-1}$

$$= \frac{1}{3}n^3 - 2n^2 + \frac{17}{3}n - (n-1) \left[ \frac{1}{3}(n^2 - 2n + 1) - 2n + 2 + \frac{17}{3} \right]$$

$$= \frac{1}{3}n^3 - 2n^2 + \frac{17}{3}n - (n-1) \left( \frac{1}{3}n^2 - \frac{8}{3}n + 8 \right)$$

simplification

$$= \frac{1}{3}n^3 - 2n^2 + \frac{17}{3}n - \frac{1}{3}n^3 + \frac{8}{3}n^2 - 8n + \frac{1}{3}n^2 - \frac{8}{3}n + 8$$

$$= n^2 - 5n + 8 //$$

3(a)

The angle between the vectors  $3\mathbf{i} - 2\mathbf{j}$  and  $6\mathbf{i} + d\mathbf{j} - \sqrt{7}\mathbf{k}$  is  $\cos^{-1}\left(\frac{6}{13}\right)$ . Find the possible value of  $d$ .

[3]

$$3\mathbf{i} - 2\mathbf{j} = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} \text{ and } 6\mathbf{i} + d\mathbf{j} - \sqrt{7}\mathbf{k} = \begin{pmatrix} 6 \\ d \\ -\sqrt{7} \end{pmatrix}$$

let  $\theta$  be the angle between the 2 vectors.

$$\cos \theta = \frac{\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ d \\ -\sqrt{7} \end{pmatrix}}{\sqrt{3^2 + (-2)^2} \cdot \sqrt{6^2 + d^2 + (-\sqrt{7})^2}}$$

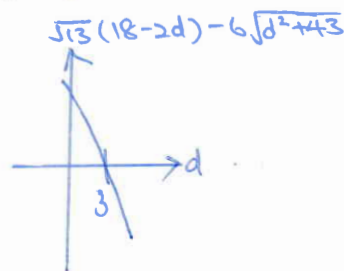
$$= \frac{18 - 2d}{\sqrt{13} \cdot \sqrt{d^2 + 43}}$$

$$\therefore \theta = \cos^{-1} \left( \frac{18 - 2d}{\sqrt{13} \sqrt{d^2 + 43}} \right) = \cos^{-1} \left( \frac{6}{13} \right)$$

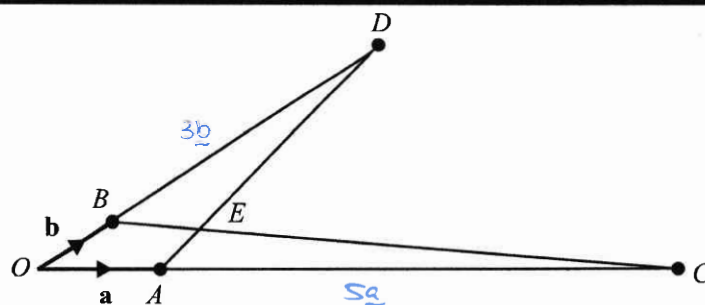
$$\Rightarrow \frac{18 - 2d}{\sqrt{13} \sqrt{d^2 + 43}} = \frac{6}{13}$$

$$\sqrt{13} (18 - 2d) = 6 \sqrt{d^2 + 43}$$

$$\text{Using the b.c., } d = 3 //$$



3(b)(i)



With reference to origin  $O$ , the points  $A, B, C$  and  $D$  are such that  $\vec{OA} = \mathbf{a}$ ,  $\vec{OB} = \mathbf{b}$ ,  $\vec{AC} = 5\mathbf{a}$  and  $\vec{BD} = 3\mathbf{b}$ . The lines  $AD$  and  $BC$  cross at the point  $E$  (see diagram).

Find  $\vec{OE}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

[5]

$$\vec{AD} = \vec{OD} - \vec{OA}$$

$$= (\mathbf{b} + 3\mathbf{b}) - \mathbf{a}$$

$$= 4\mathbf{b} - \mathbf{a}$$

Since  $E$  lies on  $AD$ ,

$$\vec{AE} = \lambda \vec{AD}, \lambda \in \mathbb{R}$$

$$\vec{OE} - \vec{OA} = \lambda (4\mathbf{b} - \mathbf{a})$$

$$\vec{OE} = 4\lambda \mathbf{b} - \lambda \mathbf{a} + \mathbf{a}$$

$$\vec{OE} = 4\lambda \mathbf{b} + (1 - \lambda)\mathbf{a} \quad \text{--- (1)}$$

$$\vec{BC} = \vec{OC} - \vec{OB}$$

$$= (\mathbf{a} + 5\mathbf{a}) - \mathbf{b}$$

$$= 6\mathbf{a} - \mathbf{b}$$

Since  $E$  lies on  $BC$ ,

$$\vec{BE} = \mu \vec{BC}, \mu \in \mathbb{R}$$

$$\vec{OE} - \vec{OB} = \mu (6\mathbf{a} - \mathbf{b})$$

$$\vec{OE} = 6\mu \mathbf{a} - \mu \mathbf{b} + \mathbf{b}$$

$$\vec{OE} = 6\mu \mathbf{a} + (1 - \mu)\mathbf{b} \quad \text{--- (2)}$$

sub (1) into (2):  $(1 - \lambda)\mathbf{a} + 4\lambda \mathbf{b} = 6\mu \mathbf{a} + (1 - \mu)\mathbf{b}$

By comparing the coefficients of vectors  $\mathbf{a}$  and  $\mathbf{b}$

$$1 - \lambda = 6\mu \Rightarrow \lambda + 6\mu = 1 \quad \text{--- (3)}$$

$$4\lambda = 1 - \mu \Rightarrow 4\lambda + \mu = 1 \quad \text{--- (4)}$$

either  
one  
correct

Using the G.C. to solve (3) and (4),

$$\lambda = \frac{5}{23}, \mu = \frac{3}{23}$$

$$\therefore \vec{OE} = 6\left(\frac{3}{23}\right)\mathbf{a} + \left(1 - \frac{3}{23}\right)\mathbf{b}$$

$$\vec{OE} = \frac{18}{23}\mathbf{a} + \frac{20}{23}\mathbf{b} //$$

3(b)(ii)

The point  $F$  on the line  $CD$  is such that  $CF:FD = 5:3$ .Show that  $O, E$  and  $F$  are collinear and find the ratio  $OE:OF$ .

[4]

$$\vec{OC} = 6\mathbf{a} \text{ and } \vec{OD} = 4\mathbf{b}$$

By Ratio Theorem,

$$\vec{OF} = \frac{5\vec{OD} + 3\vec{OC}}{5 + 3}$$

$$= \frac{1}{8}(20\mathbf{b} + 18\mathbf{a})$$

$$= \frac{23}{8}\left(\frac{18}{23}\mathbf{a} + \frac{20}{23}\mathbf{b}\right)$$

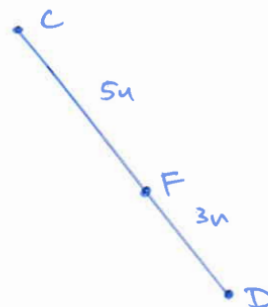
$$= \frac{23}{8}\vec{OE}$$

Since  $\vec{OF} = \frac{23}{8}\vec{OE}$  with a common point  $O$ ,  
points  $O, E$  and  $F$  are collinear. (shown) //

$$OF = \frac{23}{8}OE$$

$$\frac{OE}{OF} = \frac{8}{23}$$

$$\therefore OE:OF = 8:23 //$$



|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 4(a)(i)  | <p>Given that <math>y = \tan(e^x - 1)</math>, show that <math>\frac{dy}{dx} = ke^x(1 + y^2)</math>, where <math>k</math> is a constant to be found. [2]</p> <p> <math>y = \tan(e^x - 1)</math><br/> <math>\tan^{-1} y = e^x - 1</math><br/> Differentiate both sides implicitly w.r.t. <math>x</math>:<br/> <math>\frac{1}{1+y^2} \frac{dy}{dx} = e^x</math><br/> <math>\therefore \frac{dy}{dx} = e^x(1+y^2)</math> (shown)<br/> <math>k = 1</math> // <i>Students need to answer the value of <math>k</math>.</i> </p> <p><i>Entire working + conclusion</i></p>                                                                                                                                                                                                                                                      |
| 4(a)(ii) | <p>Hence find the values of <math>\frac{dy}{dx}</math>, <math>\frac{d^2y}{dx^2}</math> and <math>\frac{d^3y}{dx^3}</math> when <math>x = 0</math>. [4]</p> <p> <math>\frac{dy}{dx} = e^x(1+y^2)</math><br/> <math>\frac{d^2y}{dx^2} = e^x(2y) \frac{dy}{dx} + e^x(1+y^2)</math><br/> <math>= 2ye^x \frac{dy}{dx} + \frac{dy}{dx}</math><br/> <math>\frac{d^3y}{dx^3} = 2ye^x \frac{d^2y}{dx^2} + \frac{dy}{dx} (2ye^x + 2e^x \frac{dy}{dx}) + \frac{dy}{dx}</math><br/> <math>= 2ye^x \frac{d^2y}{dx^2} + \frac{dy}{dx} (2ye^x + 2e^x + 1)</math> </p> <p>When <math>x = 0</math>,</p> <p> <math>y = \tan(e^0 - 1) = \tan(0) = 0</math>.<br/> <math>\frac{dy}{dx} = e^0(1+0^2) = 1</math> //<br/> <math>\frac{d^2y}{dx^2} = 0 + 1 = 1</math> //<br/> <math>\frac{d^3y}{dx^3} = 0 + (1)(0 + 2e^0 + 1) = 3</math> // </p> |

4(b)

Write down the first three non-zero terms in the Maclaurin series for  $\tan(e^x - 1)$ .

[1]

$$\begin{aligned}\tan(e^x - 1) &= x + \frac{1}{2!}x^2 + \frac{3}{3!}x^3 + \dots \\ &= x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \dots // \end{aligned}$$

4(c)

The first two non-zero terms in the Maclaurin series for  $\tan(e^x - 1)$  are twice as big as the first two non-zero terms in the series expansion of  $e^{ax} \ln(1 + nx)$ . By using appropriate expansions from the list of formulae, find the constants  $a$  and  $n$ . Hence find the third non-zero term of the series expansion of  $e^{ax} \ln(1 + nx)$  for these values of  $a$  and  $n$ . [4]

$$\begin{aligned}e^{ax} \ln(1 + nx) &= \left(1 + ax + \frac{a^2x^2}{2} + \dots\right) \left(nx - \frac{n^2x^2}{2} + \frac{n^3x^3}{3} + \dots\right) \\ &= nx - \frac{n^2x^2}{2} + anx^2 + \frac{n^3x^3}{3} - \frac{an^2x^3}{2} + \frac{a^2nx^3}{2} + \dots \\ &= nx + \left(an - \frac{n^2}{2}\right)x^2 + \left(\frac{n^3}{3} - \frac{an^2}{2} + \frac{a^2n}{2}\right)x^3 + \dots\end{aligned}$$

By comparing the coefficients,

$$1 = 2n \Rightarrow n = \frac{1}{2} //$$

$$\begin{aligned}\frac{1}{2} &= 2\left(an - \frac{n^2}{2}\right) \\ \times 2 \quad \left( \begin{aligned} 1 &= 4a\left(\frac{1}{2}\right) - 2\left(\frac{1}{2}\right)^2 \end{aligned} \right) \times 2 \\ 1 &= 2a - \frac{1}{2}\end{aligned}$$

$$1 = 2a - \frac{1}{2}$$

$$2a = \frac{3}{2}$$

$$a = \frac{3}{4} //$$

Third non-zero term of  $e^{ax} \ln(1 + nx)$

$$\begin{aligned}&= \left[\left(\frac{1}{2}\right)^3\left(\frac{1}{3}\right) - \left(\frac{3}{4}\right)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) + \left(\frac{3}{4}\right)^2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\right]x^3 \\ &= \frac{17}{192}x^3 // \end{aligned}$$

Also accept

$$nx + \left(an - \frac{n^2}{2}\right)x^2 + \dots$$

## Section B: Probability and Statistics [60 marks]

5(a)

This question is about six couples. Each couple consists of a husband and a wife. The 12 people visit a theatre, and sit in a row of 12 seats.

In how many different ways can the 12 people sit so that each husband and wife in a couple sit next to each other? [2]

either one of these two expression seen.

$$\text{Total number of ways} = 6! \times (2!)^6 = 46080 \text{ ways}$$

$\nearrow$  no. of ways to arrange 6 couples       $\nearrow$  no. of ways each of the 6 couples can be seated HW or WH.

5(b)

In how many ways can the 12 people sit so that the 6 wives all sit next to each other, and exactly 2 of the wives sit next to their husbands. [3]

$\overbrace{H A W A W W W W}^{A \text{ and } B \text{ can swap places}} \overbrace{B H B}^{B \text{ and } H \text{ can swap places}} H H H H$

$$\text{No. of ways} = 5! \times {}^6C_2 \times 2 \times 4! = 86400 \text{ ways}$$

$\nearrow$  arrangement of the large group and 4 remaining Hs       $\nearrow$  choose 2 out of the 6 Ws to sit at the two ends. let's call them couple A and B.       $\nearrow$  couples A and B can swap places       $\nearrow$  arrangement for the remaining 4 x Ws within the large group.

1 mark.

Alternatively:  $5! \times 6! = 86400$

Arrangement of the 6 Ws in the large group.

Note: In this case,  $6!$  will determine how the 6 Ws are arranged, and there is only 1 way to seat the husbands of the two wives on each end.

5(c)

The group decides to form a committee to arrange future outings. The committee will consist of 3 of the 12 people.

In how many ways can the committee be formed without restriction?

[1]

$$\text{No. of ways} = {}^{12}C_3 = 220 \text{ ways} //$$

5(d)

In how many ways can the committee be formed such that at least 1 of the wives will be on the committee but no husband and wife couple will be included?

[3]

$$\text{No. of ways only husbands in the committee} = {}^6C_3 = 20 //$$

No. of ways a husband and wife is in the committee

$$= {}^6C_1 \times {}^{10}C_1 = 60 //$$

↑  
choose 1 out  
of 6 couples  
to be in the  
committee

↑  
choose 1 person  
out of the remaining  
5 couples (i.e. 10 people)  
to be in the committee

Alternatively,

Case 1 (1 wife, 2 other husbands):

$${}^6C_1 \times {}^5C_2 = 60$$

Case 2 (2 wives, 1 other husband):

$${}^6C_2 \times {}^4C_1 = 60$$

Case 3 (3 wives):

$${}^6C_3 = 20$$

∴ Total

$$= 60 + 60 + 20$$

$$= 140 \text{ ways} //$$

No. of ways such that at least 1 of wives will be on the committee but no husband and wife couple will be included

$$= 220 - 20 - 60 \quad (\text{complement method})$$

$$= 140 \text{ ways} //$$

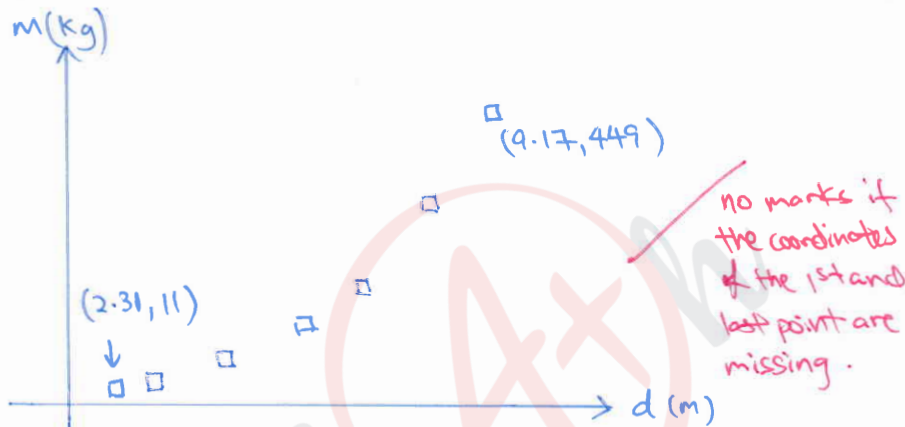
6(a)(i)

Giant pumpkins are often irregular in shape. Growers of giant pumpkins measure the size of a pumpkin using the 'over the top' length. Pumpkin growers keep records so that they can estimate the mass of giant pumpkins while they are still growing. The over the top lengths ( $d$  m) and the masses ( $m$  kg) of a random sample of 7 giant pumpkins are as follows.

|     |      |      |      |      |      |      |      |
|-----|------|------|------|------|------|------|------|
| $d$ | 2.31 | 2.90 | 4.05 | 5.50 | 6.70 | 7.92 | 9.17 |
| $m$ | 11   | 14   | 47   | 104  | 170  | 282  | 449  |

Draw a scatter diagram of the data.

[1]



6(a)(ii)

Explain how you know from your diagram that the relationship between  $m$  and  $d$  should not be modelled by a linear equation involving  $m$  and  $d$ .

[1]

From the scatter diagram, the mass appears to be increasing at an increasing rate as the top length increases. Hence, the relationship between  $m$  and  $d$  is non-linear and should not be modelled by a linear equation involving  $m$  and  $d$ .

6(b)

Which of the two equations  $m = ed^2 + f$  and  $m = gd^3 + h$ , where  $e, f, g$  and  $h$  are constants, is the better model for the relationship between  $m$  and  $d$ ? Explain fully how you decided and find the constants for the better equation.

[5]

$$\underline{m = ed^2 + f} \quad \underline{m = gd^3 + h}$$

$$r = 0.98889 \text{ (5sf)} \quad r = 0.99951 \text{ (5sf)}$$

The  $m$  in both models will increase at an increasing rate as  $d$  increases when  $e > 0$  and  $g > 0$ . However, the product moment correlation coefficient  $r$  for  $m = gd^3 + h$  is closer to 1 than that of  $m = ed^2 + f$ , suggesting a stronger positive linear relationship between  $m$  and  $d^3$ . Hence,  $m = gd^3 + h$  is a better model. //

The least squares regression line for the better model is  $m = 0.572d^3 + 3.74$  (from G.C.)  $\therefore g = 0.572, h = 3.74$  (3sf.)

1m for explanation

|            |                                                                                                                                                                                                                                                                                                                                                       |
|------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 6(c)(i)(A) | <p>Use the equation you chose from part (b) to estimate the mass of a giant pumpkin with over the top length 6 m, [1]</p> <p>when <math>d=6</math>,</p> $m = 0.57165(6)^3 + 3.7431$ $= 127 \text{ (3 s.f.)}$ <p>mass of the giant pumpkin is about 127 kg.</p>                                                                                        |
| 6(c)(i)(B) | <p>over the top length 12 m. [1]</p> <p>when <math>d=12</math>,</p> $m = 0.57165(12)^3 + 3.7431$ $= 992 \text{ (3 s.f.)}$ <p>Mass of this giant pumpkin is about 992 kg.</p>                                                                                                                                                                          |
| 6(c)(ii)   | <p>Explain which of your two estimates is more reliable. [1]</p> <p>The estimate in A is more reliable because the top length is in the data range between 2.31 and 9.17 m (interpolation), whereas the top length in B is outside of the data range (extrapolation). Hence, the estimate in A is more reliable.</p> <p>explanation + conclusion.</p> |

|      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 7(a) | <p>'Crunchers' are sweets that are sold in packets of 8. Each packet is made up of randomly chosen coloured sweets. On average 15% of Crunchers are yellow.</p> <p>Find the probability that a randomly chosen packet of Crunchers contains no more than one yellow sweet. [2]</p> <p>Let <math>Y</math> be the random variable for the no. of yellow sweets in a packet of 8 sweets. <math>Y \sim B(8, 0.15)</math></p> <p><math>P(Y \leq 1) = 0.657</math> (3 s.f.) (from G.C.)</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| 7(b) | <p>Kev buys 70 randomly chosen packets of Crunchers. Find the probability that at least 50 of these packets contain no more than one yellow sweet. [3]</p> <p>Let <math>X</math> be the random variable for the no. of packets of Crunchers containing no more than one yellow sweet, out of 70.</p> <p><math>X \sim B(70, 0.65718)</math> (5 s.f.)</p> <p><math>P(X \geq 50) = 1 - P(X \leq 49)</math></p> <p><math>= 0.190</math> (3 s.f.) (from G.C.)</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| 7(c) | <p>On average the proportion of Crunchers that are red is <math>p</math>. It is known that the modal number of red sweets in a packet is 3. [4]</p> <p>Use this information to find exactly the range of values that <math>p</math> can take.</p> <p>Let <math>R</math> be the random variable for the no. of red sweets in a packet of 8 sweets. <math>R \sim B(8, p)</math></p> <p>Since modal number of red sweets is 3</p> <div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p><math>P(R=2) &lt; P(R=3)</math>      and      <math>P(R=4) &lt; P(R=3)</math></p> <p><math>\binom{8}{2} p^2 (1-p)^6 &lt; \binom{8}{3} p^3 (1-p)^5</math></p> <p><math>\cancel{28} p^2 (1-p)^6 &lt; \cancel{56} p^3 (1-p)^5</math></p> <p><math>1-p &lt; 2p</math> (since <math>0 &lt; p &lt; 1</math>)</p> <p><math>1 &lt; 3p</math></p> <p><math>p &gt; \frac{1}{3}</math></p> </div> <div style="width: 45%;"> <p><math>\binom{8}{4} p^4 (1-p)^4 &lt; \binom{8}{3} p^3 (1-p)^5</math></p> <p><math>\cancel{70} p^4 (1-p)^4 &lt; \cancel{56} p^3 (1-p)^5</math></p> <p><math>5p &lt; 4(1-p)</math> (since <math>0 &lt; p &lt; 1</math>)</p> <p><math>9p &lt; 4</math></p> <p><math>p &lt; \frac{4}{9}</math></p> <p>either <math>p &gt; \frac{1}{3}</math><br/>or <math>p &lt; \frac{4}{9}</math> obtained</p> </div> </div> <p><math>\therefore \frac{1}{3} &lt; p &lt; \frac{4}{9}</math></p> |

**8(a)**

A bag contains 3 blue discs, 2 red discs and  $y$  yellow discs. Li chooses 3 discs at random from the bag, without replacement.

Show that the probability Li chooses 1 blue disc, 1 red disc and 1 yellow disc is  $\frac{36y}{(y+5)(y+4)(y+3)}$ . [1]

$$\text{Total no. of disc} = 3 + 2 + y = y + 5$$

$$\text{required probability} = \frac{3}{y+5} \times \frac{2}{y+4} \times \frac{y}{y+3} \times 3!$$

$$= \frac{36y}{(y+5)(y+4)(y+3)} \quad (\text{shown})$$

must include correct workings

**8(b)**

Li's discs are replaced in the bag.

Darvina chooses 3 discs at random from the bag, without replacement. The random variable  $S$  is the sum of the number of blue discs chosen and twice the number of red discs chosen.

Find an expression in terms of  $y$  for  $P(S = 3)$ .

[2]

$$P(S=3) = P(\text{picked 3 blue discs}) + P(\text{picked 1 red, 1 blue and 1 yellow})$$

$$= \frac{3}{y+5} \times \frac{2}{y+4} \times \frac{1}{y+3} + \frac{36y}{(y+5)(y+4)(y+3)}$$

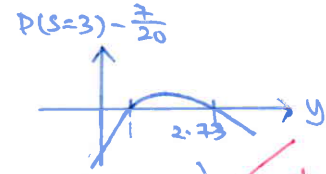
$$= \frac{6 + 36y}{(y+5)(y+4)(y+3)}$$

8(c)

Given that  $P(S=3) = \frac{7}{20}$ , calculate the value of  $y$ . Hence find the probability distribution of  $S$ .

[6]

$$\frac{6+36y}{(y+5)(y+4)(y+3)} = \frac{7}{20}$$

Using the G.C.,  $y=1$  or  $y=2.73$  (3 s.f.)(reject, since  $y$  is not a whole no.)

3B 2R 1Y  
+1 +2 +0 pt.

$$P(S=2) = P(\text{picked 2 blues and 1 yellow disc})$$

$$= \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} \times \frac{3!}{2!} = \frac{3}{20}$$

$$P(S=4) = P(\text{picked 2 reds and 1 yellow}) + P(\text{picked 2 blues and 1 red})$$

$$= \left( \frac{2}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{3!}{2!} \right) + \left( \frac{3}{6} \times \frac{2}{5} \times \frac{2}{4} \times \frac{3!}{2!} \right) = \frac{7}{20}$$

$$P(S=5) = P(\text{picked 2 reds and 1 blue})$$

$$= \frac{2}{6} \times \frac{1}{5} \times \frac{3}{4} \times \frac{3!}{2!} = \frac{3}{20}$$

| $s$      | 2              | 3              | 4              | 5              |
|----------|----------------|----------------|----------------|----------------|
| $P(S=s)$ | $\frac{3}{20}$ | $\frac{7}{20}$ | $\frac{7}{20}$ | $\frac{3}{20}$ |

(optional)

9(a)

In this question you should state the parameters of any normal distribution you use.

A type of metal bolt is manufactured with a nominal radius of 0.8 cm. In fact, the radii of the bolts, measured in cm, have the distribution  $N(0.8, 0.01^2)$ .

Find the percentage of bolts that have a radius between 0.79 cm and 0.82 cm.

[1]

Let  $X$  be the random variable for the radius of a bolt.

$$X \sim N(0.8, 0.01^2)$$

$$P(0.79 < X < 0.82) = 0.819 \text{ (3 s.f.) (from G.C.)}$$

$$\therefore \text{required percentage} = 81.9\%$$

9(b)

Metal washers are manufactured to fit on the bolts. The inside radii of the washers, measured in cm, have the distribution  $N(0.81, 0.012^2)$ .

Write down the distribution of the inside circumference of the washers, in cm, and find the circumference that is exceeded by 5% of the washers.

[4]

Let  $W$  be the random variable for the inside radius of a washer.

$$W \sim N(0.81, 0.012^2)$$

$$\text{Let } C = 2\pi W, \therefore C \sim N(2\pi(0.81), (2\pi)^2 \times 0.012^2)$$

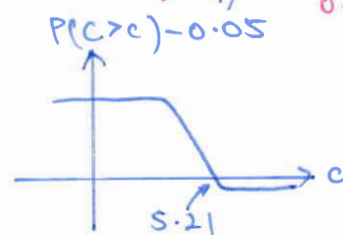
$$\Rightarrow C \sim N(1.62\pi, 0.000576\pi^2)$$

accept  
 $N(5.09, 0.00568)$

$$\text{Let } P(C > c) = 0.05$$

$$P(C > c) - 0.05 = 0$$

$$\text{Using the G.C.}, c = 5.21 \text{ (3 s.f.)}$$



$\therefore$  The circumference that is exceeded by 5% of the washers is 5.21 cm (3 s.f.)

9(c)

A bolt and a washer are a 'good fit' if

- the inside radius of the washer is greater than the radius of the bolt and
- the inside radius of the washer is not more than 0.04 cm greater than the radius of the bolt.

A washer is chosen at random and a bolt is chosen at random. Find the probability that the washer and bolt are a good fit. [3]

$$W - X \sim N(0.81 - 0.8, 0.012^2 + 0.01^2)$$

$$\Rightarrow W - X \sim N(0.01, 0.000244)$$

$$P(0 < W - X \leq 0.04) = 0.712 \text{ (3s.f.) (from G.C.)}$$

9(d)

The outside radii of the washers, measured in cm, have the distribution  $N(\mu, \sigma^2)$ . It is known that 15% of the washers have an outside radius greater than 1.25 cm and 25% have an outside radius of less than 1.15 cm.

Find the values of  $\mu$  and  $\sigma$ . [3]

Let  $Y$  be the random variable for the outside radius of a washer.

$$Y \sim N(\mu, \sigma^2)$$

$$P(Y > 1.25) = 0.15$$

$$P\left(\frac{Y - \mu}{\sigma} > \frac{1.25 - \mu}{\sigma}\right) = 0.15$$

$$P\left(Z > \frac{1.25 - \mu}{\sigma}\right) = 0.15$$

Using Inv Norm on the G.C.,

$$\frac{1.25 - \mu}{\sigma} = 1.0364 \text{ (5s.f.)}$$

$$\mu + 1.0364\sigma = 1.25 \text{ --- (1)}$$

Solving (1) and (2) using the G.C.,

$$\mu = 1.19 \quad \sigma = 0.0584 \text{ (3s.f.)}$$

$$P(Y < 1.15) = 0.25$$

$$P\left(\frac{Y - \mu}{\sigma} < \frac{1.15 - \mu}{\sigma}\right) = 0.25$$

$$P\left(Z < \frac{1.15 - \mu}{\sigma}\right) = 0.25$$

$$\text{where } Z = \frac{Y - \mu}{\sigma} \sim N(0, 1)$$

Using the Inv Norm on the G.C.

$$\frac{1.15 - \mu}{\sigma} = -0.67449 \text{ (5s.f.)}$$

$$\mu - 0.67449\sigma = 1.15 \text{ --- (2)}$$

both  $\mu$  and  $\sigma$  correct.

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 10(a)    | <p>The average time required for the manufacture of a certain type of electronic control panel is 17 hours. An alternative manufacturing process is trialled, and the time taken, <math>t</math> hours, for the manufacture of each of 50 randomly chosen control panels using the alternative process is recorded. The results are summarised as follows.</p> $n = 50 \quad \Sigma t = 835.7 \quad \Sigma t^2 = 14\,067.17$ <p>The Production Manager wishes to test whether the average time taken for the manufacture of a control panel is different using the alternative process, by carrying out a hypothesis test.</p> <p>Explain whether the Production Manager should use a 1-tail test or a 2-tail test. [1]</p> <p>The Production Manager should use a 2-tail test since he/she wishes to test if the average time is different, which could either be more than or less than 17 hours.</p> |
| 10(b)    | <p>Explain why the Production Manager is able to carry out a hypothesis test without knowing anything about the distribution of the times taken to manufacture the control panels. [1]</p> <p>Since the <u>sample size is large</u> (i.e. <math>50 &gt; 30</math>), by <u>Central Limit Theorem</u>, the sample mean time taken to manufacture the control panels using the alternative process would follow a <u>Normal Distribution approximately</u>.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| 10(c)(i) | <p>Find unbiased estimates of the population mean and variance. [2]</p> <p>unbiased estimate of the population mean = <math>\frac{835.7}{50} = 16.714</math> (exact)</p> <p>unbiased estimate of the population variance</p> $= \frac{1}{49} \left( 14\,067.17 - \frac{835.7^2}{50} \right)$ $= 2.0261 \text{ (5 s.f.)}$ $= \underline{\underline{2.03}} \text{ (3 s.f.)}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |

|                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|-----------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>10(c)(ii)</p> <p>Define Im</p> | <p>Carry out the test at the 10% level of significance for the Production Manager. Define any symbols you use. [5]</p> <p>Let <math>T</math> be the time taken in hours for the manufacture of a randomly chosen control panel using the alternative process.</p> <p>Let <math>\mu</math> be the population mean time taken to manufacture these panels.</p> <p>Test <math>H_0: \mu = 17</math><br/>against <math>H_1: \mu \neq 17</math> at 10% significance level</p> <p><math>H_0</math> and <math>H_1</math> are the null and alternative hypothesis respectively.</p> <p>Under <math>H_0</math>, since sample size is large (i.e. <math>50 &gt; 30</math>), by Central Limit Theorem, <math>\bar{T} \sim N(17, \frac{2.0261}{50})</math> approximately.</p> <p>Test statistics, <math>Z_{\text{test}} = \frac{16.714 - 17}{\sqrt{2.0261/50}} = -1.4208</math> (5sf)</p> <p>For 2-tailed test, critical value <math>\pm Z_{0.05} = \pm 1.6449</math> (from G.C.)</p> <p>Since <math>Z_{\text{test}} = -1.4208 &gt; -1.6449</math>, we do not reject <math>H_0</math>, and conclude that there is insufficient evidence, at 10% significance level, that the time taken for the manufacture of the control panels is different using the alternative process.</p> <p>correct <math>H_0</math> and <math>H_1</math></p> <p>Define distribution</p> <p>comparison. Alternatively, accept p-value = 0.15539 &gt; 0.01</p> <p>correct conclusion</p>                                                         |
| <p>10(d)</p> <p>Im</p>            | <p>The Finance Manager wishes to test whether the average time taken for the manufacture of a control panel is shorter using the alternative process. The Finance Manager finds that the average time taken for the manufacture of each of 40 randomly chosen control panels, using the alternative process, is 16.7 hours. He carries out a hypothesis test at the 10% level of significance.</p> <p>Given that the Finance Manager concludes that there is no reason to reject the null hypothesis, find the range of possible values of the population variance used in calculating the test statistic. [3]</p> <p>Let <math>\sigma^2</math> be the population variance used in calculating the test statistic.</p> <p>Test <math>H_0: \mu = 17</math><br/>against <math>H_1: \mu &lt; 17</math> at 10% significance level.</p> <p>Under <math>H_0</math>, since sample size is large (i.e. <math>40 &gt; 30</math>), by Central Limit Theorem, <math>\bar{T} \sim N(17, \frac{\sigma^2}{40})</math> approximately.</p> <p>Test statistic, <math>Z_{\text{test}} = \frac{16.7 - 17}{\sigma/\sqrt{40}}</math></p> <p>To not reject <math>H_0</math>, <math>Z_{\text{test}} &gt; -Z_{0.1}</math> where <math>P(Z &lt; -Z_{0.1}) = 0.1</math>, <math>Z \sim N(0,1)</math></p> <p><math>\frac{16.7 - 17}{\sigma/\sqrt{40}} &gt; -1.2816</math> (5sf)</p> <p><math>\sqrt{40}(-0.3) &gt; -1.2816\sigma \Rightarrow \sigma &gt; 1.4805</math> (5sf)</p> <p><math>\therefore \sigma^2 &gt; 2.19</math> (3sf)</p> |